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A METHOD FOR CALCULATING THREE-DIMENSIONAL FLOW IN BLADED TURBOMACHINES BY USING ARBITRARY QUASI PERPENDICULAR INTERSECTING PLANES

by

Xin Xiaokang, Jiang Jinliang

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A METHOD FOR CALCULATING THREE-DIMENSIONAL FLOW IN BLADED TURBOMACHINES BY USING ARBITRARY QUASI PERPENDICULAR INTERSECTING PLANES

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Since the Great Proletariat Cultural Revolution, considerable development has been seen in the research, design and manufacture of bladed turbomachines in China. To further increase the efficiency and power output with small dimensions, at present the theory of three-dimensional fluid flow is being applied to upgrade design calculations in many plants and organizations of scientific research. We follow Great Leader Chairman Mao's instruction, "Revolution in Education" by meeting the demands of industrial and agricultural production as a research subject. Along with the Shanghai Shipyard, Shanghai Diesel Engine Plant, and Shanghai Institute of Internal Combustion Engines, we are engaged in the design and calculation of three-dimensional fluid flow in superchargers. This paper presents one result in our open-door educational activity.

Reports (1,2) first proposed fundamental equations and calculation processes in the three-dimensional fluid flow in bladed turbomachines. This calculation method aims at simplifying the three-dimensional flow field in a bladed turbomachine into two two-dimensional flow fields (S_1 and S_2 flow surfaces); by mutually successive substitutions of these two two-dimensional flow fields,

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Translator's note: flow surface or surface of flow should read throughflow surface.

solutions of three-dimensional flow field can be obtained. Even at the present time, this calculation method is widely adopted both domestically and abroad. However, because a great volume of calculations is required to mutually and successively substitute between the two groups of flow surfaces, the present calculation method (3-10) of three-dimensional fluid flow does not employ successive substitutions. Instead, one successive substitution from the S_2 to S_1 flow surface is used to derive the quasi three-dimensional solution.

From the method of arbitrary quasi perpendicular intersecting lines (6-8) in the streamline curvature method, we developed a method of arbitrary quasi perpendicular intersecting planes. In calculations, the mutual influence between the two groups of flow surfaces is simultaneously considered, so that no corrections need to be made from each other by successive substitution between these two groups of flow surfaces. In this method, the theory of a variable curve sample is applied. After appropriate successive substitutions of spatial streamlines, the three-dimensional solution of the entire blade channel can be directly calculated, including shapes of arbitrary S_1 and S_2 flow surfaces, shapes of spatial streamlines, and the distribution of flow parameters, such as velocity and pressure.

As the first step in solving the real three-dimensional flow field, we apply the approximate hypothesis that the S_1 flow surface is a surface of revolution; this hypothesis is applied in most reports. Thus, there are briefer and more convenient steps in flow integration and reverse interpolation of equal-sector flow into new streamlines. Hence, the solution obtained is basically still the quasi three-dimensional solution. The next step is to eliminate the hypothesis that the S_1 surface is a surface of revolution in order to obtain the solution of a real three-dimensional flow field. Then the curvature of S_1 flow surface can be calculated.

I. The Method of Arbitrary Quasi Perpendicular Intersecting Planes

In principle, this method can be applied to axial-flow, diametrical-flow, and mixed-flow bladed turbomachines. The paper uses an example of a bladed channel in a centrifugal compressor bladed turbomachine for a detailed illustration.

Figure 1a shows a bladed channel of a centrifugal compressor bladed rotor. Figure 1b shows the shape of the meridian plane of the bladed channel. The report (6) introduces the situation of utilizing an arbitrary quasi perpendicular

interesting line q_m to solve for meridian flow field (S_2 flow surface). The report (7) introduces a situation in which use is made of an arbitrary quasi perpendicular intersecting line q_θ to solve for the trans-blade flow field (S_1 flow surface). In the ordinary method of the quasi perpendicular intersecting line, first we integrate the velocity gradient equation in the direction of q_m or q_θ. Next, by flow correction the velocity distribution along the quasi perpendicular intersecting line can be obtained. Then, by inverse interpolation of equal-sector flows, the position of new streamlines at the S_2 or S_1 flow surface can be obtained. In the paper, the quasi perpendicular intersecting lines can constitute a curved surface S_q (See Fig. 1a), which is formed by the quasi perpendicular interescting straight line q_m (Fig. 1b) revolving around the z axis for a complete circle. We then take this curved surface as the quasi perpendicular intersecting plane in solving the problem. First, the fundamental equations can be used to derive a velocity gradient equation along an arbitrary direction in space. Then two velocity gradient equations can be derived along the q_m and q_θ directions. Integrate these two equations. Next, through flow correction, the velocity distribution at the quasi perpendicular intersecting plane can be obtained. Then, by reverse interpolation of equal-sector flow, new positions of spatial streamlines can be obtained.

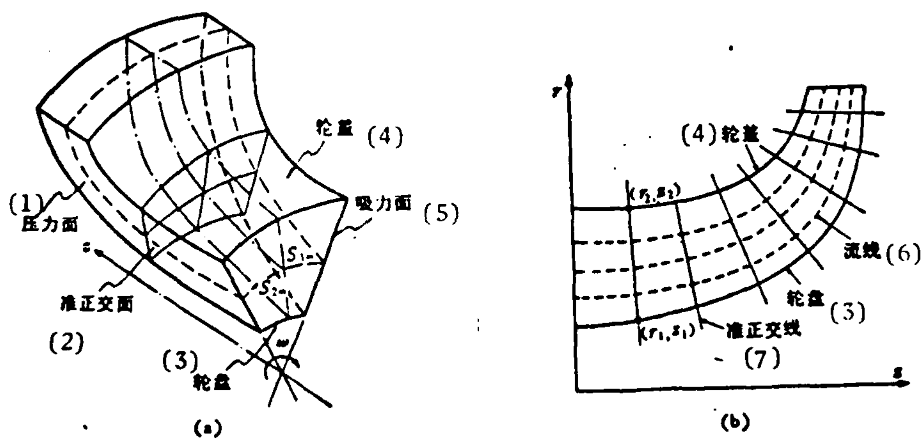

Fig. 1.
Key: (1) Pressure surface; (2) Quasi perpendicular intersecting plane; (3) Rotor; (4) Stator; (5) Suction surface; (6) Streamline; (7) Quasi perpendicular intersecting line.

The fundamental hypothesis adopted by the paper is as follows:

1. The working medium (or fluid) is a perfect gas, which is viscousless with equal C_p. The flow is isentropic. Friction losses in a real gas can be adjusted by total pressure loss as indicated in reports (6,7), or revealed by using the

ordinary polytropic process.

2. The flow passing through the blade rotor is of finite length, relative to blade rotor.

3. The S_1 flow surface is a surface of revolution. This hypothesis is identical to the hypothesis used in most reports, and restricts the application range of this method.

We now proceed to derive the fundamental equations used in the paper.

In reports (6,7), the velocity gradient equation at any curve direction in the space along the flow field is as follows:

$$\frac{dW}{dq} = a\frac{dr}{dq} + b\frac{dz}{dq} + c\frac{d\theta}{dq} + \frac{1}{W}\left(\frac{dh_i'}{dq} - \omega\frac{d\lambda}{dq}\right) \tag{1}$$

In equation (1),

$$\left.\begin{aligned} a &= \frac{W\cos\alpha\cos^2\beta}{r_c} - \frac{W\sin^2\beta}{r} + \sin\alpha\cos\beta\frac{dW_m}{dm} - 2\omega\sin\beta \\ b &= -\frac{W\sin\alpha\cos^2\beta}{r_c} + \cos\alpha\cos\beta\frac{dW_m}{dm} \\ c &= W\sin\alpha\sin\beta\cos\beta + r\cos\beta\left(\frac{dW_\theta}{dm} + 2\omega\sin\alpha\right) \end{aligned}\right\} \tag{2}$$

Here, W is the relative velocity; q is the intercept of the space curve; r_c is the radius of curvature of the projection by the streamline on the meridian plane (briefly called the meridian streamline); α is the included angle between the z axis (rotating axis) and the tangent of the meridian streamline; β is the included angle (Fig. 2a,b) between the meridian plane and the relative velocity vector; λ is the previous rotation ($\lambda=rV_\theta$); h_i' is the total entropy of each streamline; and ω is the angular velocity of the blade rotor rotation.

From Fig. 2, the following geometric relationship equation can be established:

$$
\left.
\begin{array}{ll}
W^2 = W_m^2 + W_\theta^2, & W_m^2 = W_r^2 + W_z^2, \\
W_r = W_m \sin\alpha, & W_z = W_m \cos\alpha, \\
W_m = W \cos\beta, & W_\theta = W \sin\beta, \\
\operatorname{tg}\beta = \dfrac{r\,d\theta}{dm} &
\end{array}
\right\} \qquad (3)
$$

Here, W_r, W_θ, and W_z are, respectively, the relative velocity components along the diametrical direction, circumferential direction, and axial direction; W_m is the relative velocity component of meridian plane; dm is the micro-element of the arc length of the meridian streamline.

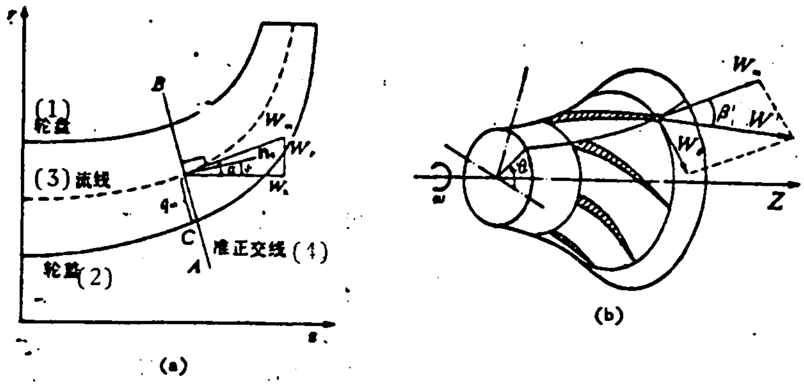

Fig. 2
Key: (1) Rotor; (2) Stator; (3) Streamline; (4) Quasi perpendicular intersecting line.

In the appendix, velocity gradient equations can be derived in the direction of two groups of curves along the quasi perpendicular intersecting plane:

One velocity gradient equation is in the direction of the intersecting line between the quasi perpendicular intersecting plane and the blade pressure surface

$$
\frac{dW}{dq_m} = AW + B + (CW + D)\frac{d\theta_e}{dq_m} + \frac{1}{W}\left(\frac{dh_i'}{dq_m} - \omega\frac{d\lambda}{dq_m}\right) \qquad (4)
$$

In Equation (4),

$$
\left.
\begin{array}{l}
A = \dfrac{\cos^2\beta\cos(\psi-\alpha)}{r_e} - \dfrac{\sin^2\beta\cos\psi}{r} \\
B = -\cos\beta\sin(\psi-\alpha)\dfrac{dW_m}{dm} \\
\qquad - 2\omega\sin\beta\cos\psi \\
C = \sin\alpha\sin\beta\cos\beta \\
D = r\cos\beta\left(\dfrac{dW_\theta}{dm} + 2\omega\sin\alpha\right)
\end{array}
\right\} \qquad (5)
$$

Here, ψ is the included angle between the z axis and the normal line $\bar{n}_q$ of the intersecting line AB referred to concisely as the quasi perpendicular intersecting line between the meridian plane and the quasi perpendicular intersecting plane; q_m is a parameter, indicating the intercept along line AB starting from the intersecting point c between line AB and the rotor; $\theta_d = f_d(r,z)$ showing the pressure surface equation of the blade.

Another velocity gradient equation is in the arc direction (θ direction) when q_m=constant along the quasi perpendicular intersecting plane.

$$\frac{dW}{d\theta} = CW + D + \frac{1}{W}\left(\frac{dh_i'}{d\theta} - \omega\frac{d\lambda}{d\theta}\right) \qquad (6)$$

Here, C and D are the same as in Equation (5).

The continuous equation is satisfied by using the same flow (equivalent to a given flow) passing through the quasi perpendicular intersecting plane. The following is the equation of flow weight as the flow passes through an arbitrary quasi perpendicular intersecting plane and stays within the channel of two adjacent blades.

$$\begin{aligned} G &= \int_0^{q_{mB}} \int_{\theta_d}^{\theta_s} \rho g \mathbf{W} \cdot \mathbf{n} \cdot r d\theta dq_m \\ &= \int_0^{q_{mB}} \int_{\theta_d}^{\theta_s} \rho g W_m \cos(\psi - \alpha) r d\theta dq_m \end{aligned} \qquad (7)$$

In Equation (7), θ_d is the angle of the blade pressure surface and θ_s is the angle of the blade suction surface. As integrations are conducted along a curved surface, $\theta_d = \theta_d(q_m)$ and $\theta_s = \theta_s(q_m)$; these are equations of the curve.

These three equations are the fundamental equations we adopted.

II. The Solution Procedures in This Method

The procedures in this method are as follows:

1. First on the meridian plane, arbitrarily select m quasi perpendicular intersecting lines (Fig. 1b), which are quasi perpendicularly intersecting with all meridian streamlines. For sake of simplicity, generally straight lines are selected. Then the corresponding angle ψ of these m quasi perpendicular intersecting lines can be written as

$$\psi_i = \arcsin\left(\frac{z_{1i} - z_{2i}}{\sqrt{(z_{2i} - z_{1i})^2 + (r_{2i} - r_{1i})^2}}\right) \quad (i = 1, 2, \cdots, m) \qquad (8)$$

In the equation, (r_{1i}, z_{1i}) and (r_{2i}, z_{2i}) are, respectively, the coordinates of the intersecting points between the quasi perpendicular intersecting line on one side, and the rotor and stator, on the other.

2. Rotate this group of quasi perpendicular intersecting lines about the z axis, we obtain the required quasi perpendicular intersecting planes.

3. Form the initial meridian streamlines on a meridian plane. As usual, we can adopt an equidistant division of quasi perpendicular intersecting straight lines, or make the revolving-ring-shaped channel areas equal in value. We adopt the first method by dividing into n equal parts to obtain (n+1) meridian streamlines. Then we can see that the coordinates (r_B, z_B) of the intersecting point between the initial meridian streamlines and the quasi perpendicular intersecting lines are:

$$\left.\begin{aligned}(r_B)_{i,j} &= r_{1i} + \frac{j-1}{n}(r_{2i} - r_{1i})\\(z_B)_{i,j} &= z_{1i} + \frac{j-1}{n}(z_{2i} - z_{1i})\end{aligned}\right\} \quad \begin{pmatrix} i = 1, 2, \cdots, m \\ j = 1, 2, \cdots, n+1 \end{pmatrix} \tag{9}$$

4. In the hypothesis, the S_1 flow surface is a surface of revolution, therefore, as mentioned above, revolving the (n+1) meridian streamlines around z axis will generate (n+1) S_1 flow surfaces. The initial space streamlines can be generated on (n+1) S_1 flow surfaces.

First, the cutting of a blade channel of the rotor by a quasi perpendicular intersecting plane is drawn in Figure 3. Figure 3a is the developed diagram (projection on a plane with z=constant) in an ordinary rectangular coordinate system. Figure 3b is the developed diagram in the q_m-θ coordinate system.

From Figure 3, when a quasi perpendicular intersecting plane cuts through the flow channel, the boundary of the cross section thus produced forms a curved parallelogram. On q_m-θ diagram, the upper and lower sides of the parallelogram are straight lines: $q_m=0$ and $q_m=q_{m\ \mathrm{stator}}$

Moreover, the intersecting line between the S_1 flow surface and the quasi perpendicular plane is also a straight line:

$$(q_m)_1=[(j-1)/n]q_{m\ \mathrm{stator}}$$

Therefore, along these straight lines, r=constant and z=constant. The variation range of angle θ is $\theta_d \leqslant \theta \leqslant \theta_s$

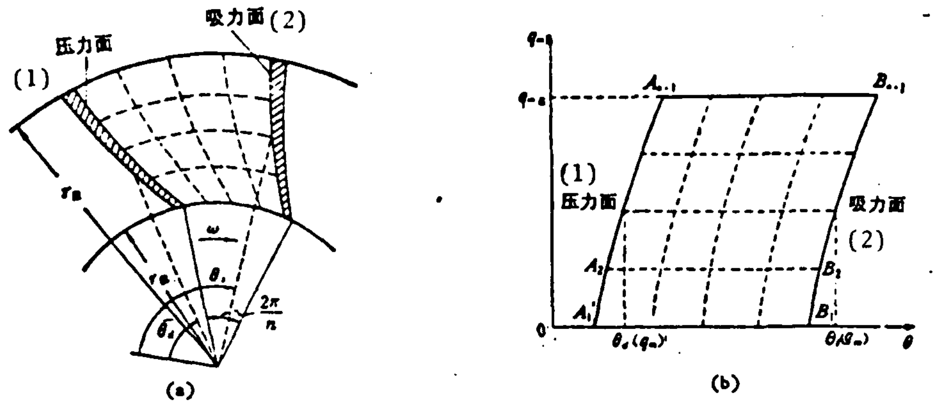

Fig. 3
Key: (1) Pressure surface; (2) Suction surface.

Now, we make h equal divisions on θ at the straight lines q_m=constant; then we have (n+1)(h+1) initial space streamlines. Then we can derive the result that the coordinates of θ of the intersecting point between the quasi perpendicular intersecting plane and the initial space streamlines are:

$$(\theta_s)_{i,j,k} = \theta_d(r_{i,j}, z_{i,j}) + \frac{k-1}{h}[\theta_s(r_{i,j}, z_{i,j}) - \theta_d(r_{i,j}, z_{i,j})]\begin{pmatrix} i = 1, 2, \cdots, m \\ j = 1, 2, \cdots, n+1 \\ k = 1, 2, \cdots, h+1 \end{pmatrix} \quad (10)$$

In the equation, $\theta_d(r,z)$ and $\theta_s(r,z)$ are determined by the blade forming equation of air-guiding rotor and working rotor. The analytical equation can be derived or values of divergent points can be obtained. Thus, after r and z are given, values of θ_d and θ_s can be solved or obtained by interpolation.

Thus, we obtain nodes of initial streamlines in the entire blade channel. There are altogether m quasi perpendicular intersecting planes, (n+1) S_1 flow surfaces and (h+1) S_2 flow surfaces. Coordinates of intersecting points (nodes) of these three groups of surfaces can refer to Equations (9) and (10).

5. After obtaining the coordinates of the initial streamlines, according to the theory of sampling curve we can obtain various geometric quantities (such as α, β, and r_c) of streamlines and solve for various parameters (such as dW_m/dm and dW_U/dm) required in Equations (4), (6) and (7). The solution procedures are

the same as in report (6). As in report (6), the initial value of W can be assumed constant; in our case, the initial value of W are assumed as constant at each quasi perpendicular intersecting plane for more rapid convergence in successive substitutions. Here, the initial value of $W_i^{(0)}$ is given by input.

6. Same equations for numerical solution can be adopted in the numerical integration of two differential equations (4) and (6). We still adopt the prediction - correction method (i.e., Runge-Kutta method) described in report (6).

In this method, the procedure for solving W values of the entire flow field is as follows: To each quasi perpendicular intersecting plane, first solve for W values (Fig. 3b) at points A_1, A_3,..., A_{n+1} with initial values of point A_1 by using pressure surface integration equation (4). Then, use q_m=constant to integrate linear differential equation (6), from point A_i to point B_i at the suction surface in order to obtain W values of the entire net lattice points.

7. Use flow conservation equation (7) to check whether the total flow can be satisfied in order to revise the initial value of W for point A_1.

$$G_k = \int_{\theta_d}^{\theta_s} \rho g W \cos\beta \cos(\psi - \alpha) d\theta \qquad (11)$$

Then the flow weight equation (7) passing through any arbitrary quasi perpendicular intersecting plane can be written as

$$G_i = \int_0^{q_m \text{ stator}} G_k(q_m) \cdot (r_1 + q_m \cos\psi) dq_m \qquad (12)$$

These integrations can be easily obtained by using the sampling curve theory of the divergent point. Moreover, a group of successively additive numbers can be obtained. If the calculated total flow G_i and the given flow G_0 do not satisfy precision requirement $|G_i - G_0| < \varepsilon_G$ (ε_G input) (13)

Then, we can use

$$W'_{A_1} = W_{A_1} \frac{G_0}{G_i} \qquad (14)$$

to be the new initial value of the velocity at point A_1. Repeat steps 6 and 7 until the precision requirement (13) is satisfied. Generally, this process requires five to six iterations.

8. After satisfying the precision requirement (13), apply the method of reverse interpolation of equal-division flow and solve for the streamline

coordinates (r_G, z_G and θ_G).

First, apply reverse interpolation by n equal divisions of meridian flow at the total flow curve G_i-q_m (Fig. 4a) and solve for various values of $(q_m)_G$, then

$$\left.\begin{aligned} r_G &= r_1 + (q_m)_G \cdot \cos\psi \\ z_G &= z_1 - (q_m)_G \cdot \sin\psi \end{aligned}\right\} \qquad (15)$$

Apply reverse interpolation to angle θ and reverse interpolation for h equal divisions at the flow curve G_k-θ (Fig. 4b), we obtain the value of ϑ_G. However, since values of r and z have changed, values of θ_d and θ_s for boundary points should start from r_G and z_G values using blade equations $\theta_d = \theta_d(r,z)$ and $\theta_s = \theta_s(r,z)$ and solving for values of $(\theta_d)_G$ and $(\theta_s)_G$. Here we employed an approximation hypothesis: we considered that the θ_G values obtained for the original r and z can be approximately regarded as the partition values for r_G and z_G but θ_d and θ_s nonetheless use the values $(\theta_d)_G$ and $(\theta_s)_G$. Because the loosening factor η is adopted later on, this approximation will not affect convergence.

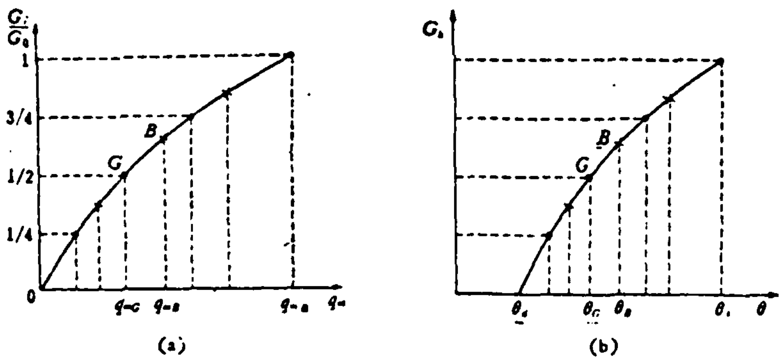

Fig. 4.

9. Lastly, we derive the maximum deviation between new and old streamline points

$$M = \max_{\substack{1\le i\le m \\ 1\le j\le n+1 \\ 1\le k\le h+1}} [(r_G - r_B)^2 + (z_G - z_B)^2 + r^2(\theta_G - \theta_B)^2] \qquad (16)$$

When $M < \varepsilon_m$ (input value), we obtain coordinate positions and velocity distribution of streamlines by using the calculation results of the printout. Otherwise, utilize the loosening factor η (input) of the streamline coordinates to obtain new streamline coordinates:

$$\left.\begin{aligned} r_B + \eta(r_G - r_B) &\Longrightarrow r_B \\ z_B + \eta(z_G - z_B) &\Longrightarrow z_B \\ \theta_B + \eta(\theta_G - \theta_B) &\Longrightarrow \theta_B \end{aligned}\right\} \qquad (17)$$

Again, return to step 5 for iterated calculations until $M<\varepsilon_m$.

Now, we make several explanatory notes about the solution process:

1. Just as for procedures in report (6), when solving for α, first rotate the r-z coordinates by 45° and then apply the method of obtaining first order derivative by using sampling curves.

2. Just as for procedures used in report (12), when solving for r_c, use two sampling curves to improve the precision with the same conditions of terminal points as in report (6).

3. During trial claculation used in the paper, for the time being the variations of h_i' and λ are left aside. This is feasible in compressor calculations of superchargers. Variations of h_i' and λ will not cause any other difficulties.

4. As for loss revisions, we may use the correction method of total pressure loss in report (6). However, for convenience this program adopts a correction to the multivariant index:

$$\frac{n_0}{n_0-1}=\eta\frac{\kappa}{\kappa-1}$$

Here, k is the adiabatic index, n_o is the multivariant index; and η is the multivariant efficiency. The value of n_o is determined by input.

III. Analysis and Discussion of Results

A program was compiled by using this method; trial calculations were conducted on a centrifugal compressor of a Number 780 supercharger designed by the Shanghai Shipyard, the Shanghai Institute of Ship Transportation and other units. Refer to Figure 5 for outlines of the meridian planes of a compressor blade rotor. The following are the performance parameters thus calculated: inlet temperature T_i=303°K; inlet specific gravity γ_i=1.149 kg/m^3; isopiestic specific heat C_p=1003.56 kg·m/kg; multivariant index n_o=1.509; flow weight G_o=24 kg; angular velocity ω=880 radians/sec; number of blades Z_n=20; number of quasi perpendicular intersecting planes m=12; number of S_1 flow surfaces (n+1)=9; number of S_2 flow surfaces (h+1)=9.

The common parabola forming method is used to generate outlines of air-guiding rotors; the working rotor has diametrical straight blades.

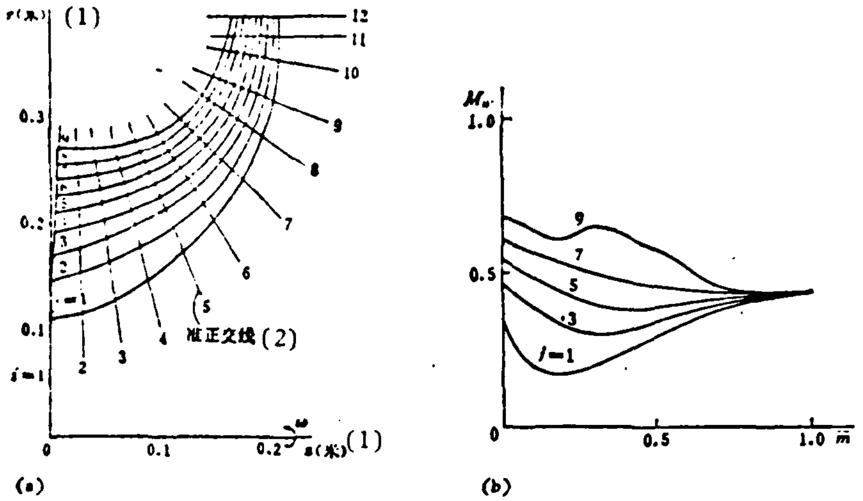

Fig. 5
(a) Shapes of Meridian Streamlines and Meridian Plane Outlines of a Blade Rotor; (b) Distribution of Relative Mach Number on the Averaged S_2 Flow Surface (k=5).
Key: (1) Meter (m); (2) Quasi perpendicular intersecting lines.

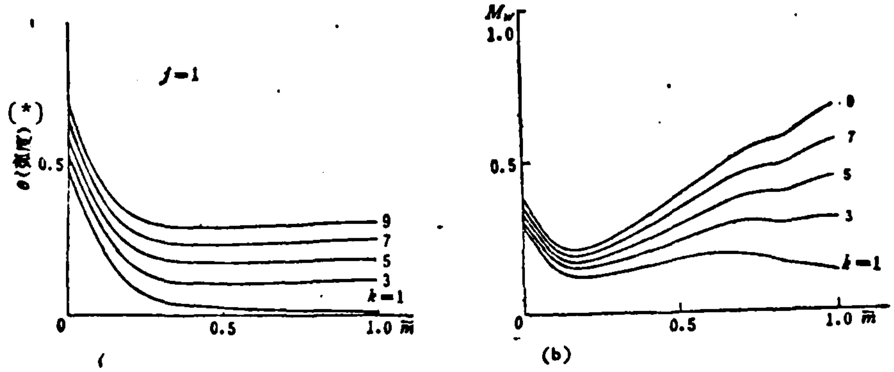

Fig. 6
(a) Streamline Shapes on a Rotor (j=1); (b) Distribution of Relative Mach Number on a Rotor (j=1).
Key: (*) Radian

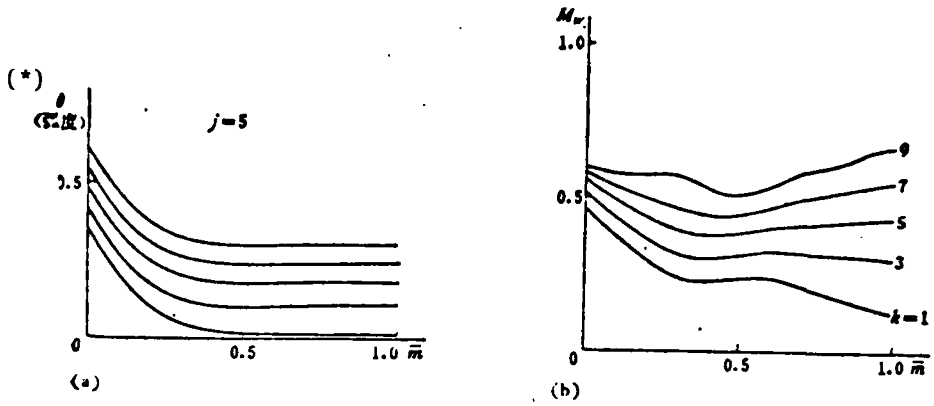

Fig. 7
(a) Streamline Shapes on an Averaged Trans-blade Surface (j=5);
(b) Distribution of Relative Mach Number on a Rotor (j=1).
Key: (*) Radian.

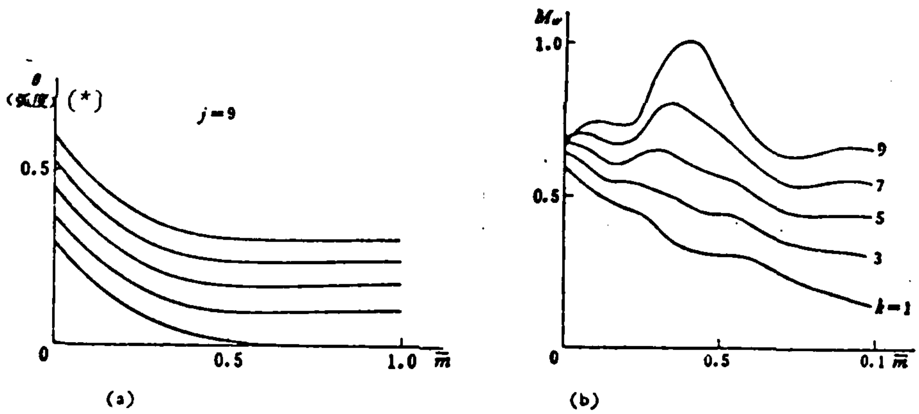

Fig. 8
(a) Streamline Distribution on a Rotor (j=9); (b) Distribution of Relative Mach Number on a Rotor (j=9).
Key: (*) Radian.

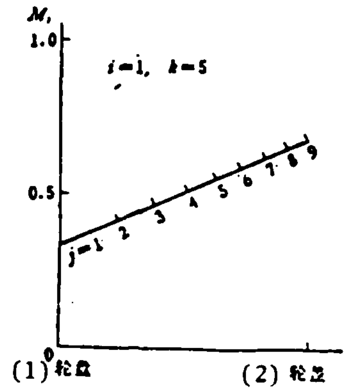

Fig. 9 Distribution of relative Mach number on a quasi perpendicular intersecting line at the inlet.
Key: (1) Rotor; (2) Stator.

The calculation results are plotted in Figures 5-9. Since there are many data, for sake of simplicity, these curves drawn are streamlines of the trans-blade flow field for j=1, 5 and 9; the corresponding velocity distributions for k=1, 3, 5, 7 and 9; the streamline distribution for k=5 (i.e., the average surface of flow S_{1m} channels); and the velocity distribution for j=1, 3, 5, 7 and 9.

From the calculation results, we drew the following primary conclusions:

1. We can see from Fig. 9 that on the average throughflow surface (k=5), the distribution of relative Mach number M_W at the inlet on the quasi perpendicular intersecting lines is quite similar to that shown in Fig. 7 of report (12). The shapes of velocity distribution and streamline plot spectra in Figs 5-8 are also consistent with curves in other reports.

2. For this example, when ε_G=1%, $\varepsilon_m=10^{-4}$ and the loosening factor η is selected as 0.1, the three-dimensional flow field solution can be obtained by applying more than 20 successive substitutions. Besides the printout time, the processing time is approximately 8-10 minutes by using our university's 719 computer (125,000 operations per second). This processing time is considerably less than the mutually successive substitutions of nine S_1 flow surfaces and nine S_2 flow surfaces in several calculations.

3. The shortcoming of this method is the same as the common streamline

curvature method: it is generally difficult to calculate precisely the flows at inlet and exit -- corrections are required. However, the revision of the three-dimensional flow field differs from the situation of solving separately the S_1 and S_2 flow surfaces. The work will be continued later on.

4. Since the method in the paper is based on the assumption that S_1 flow surface is a surface of revolution with more limitations, one still cannot calculate the curving state of the S_1 flow surface. As a continuation of the paper, later this assumption will be eliminated in order to derive a real solution of the three-dimensional flow field.

APPENDIX

In the following, we deduce Equations (4), (6) and (7).

Since the quasi perpendicular intersecting plane we select is a plane of revolution, the generating line is a straight line (AB in Fig. 2a) of a meridian plane:

$$\left.\begin{aligned} r &= r_1 + q_m \cos\psi \\ z &= z_1 - q_m \sin\psi \end{aligned}\right\} \qquad \text{(A1)}$$

In the equation, (r_1, z_1) are coordinates of intersecting point c between the blade rotor and the quasi perpendicular intersecting line AB; q_m is the intercept along line AB starting from point c; and ψ is the included angle between z axis and n_q, the normal line of AB.

By eliminating q_m in Equation (A1), the straight line equation is

$$(r - r_1)\sin\psi + (z - z_1)\cos\psi = 0 \qquad \text{(A2)}$$

If the equation of blade pressure surface is

$$\theta_d = f_d(r, z) \qquad \text{(A3)}$$

the conditions of the intersecting curve (space curve) equation between the pressure surface and the quasi perpendicular intersecting plane are

$$\left.\begin{aligned} r_d &= r_1 + q_m \cos\psi \\ z_d &= z_1 - q_m \sin\psi \\ \theta_d &= f_d(r_d, z_d) = \theta_d(q_m) \end{aligned}\right\} \qquad \text{(A4)}$$

The direction of q in Equation (1) takes on the direction of intersecting curve, then

$$\frac{dW}{dq_m} = a\frac{dr_d}{dq_m} + b\frac{dz_d}{dq_m} + c\frac{d\theta_d}{dq_m} + \frac{1}{W}\left(\frac{dh'_i}{dq_m} - \omega\frac{d\lambda}{dq_m}\right) \qquad \text{(A5)}$$

From Equation (A5), we know

$$\frac{dr_d}{dq_m} = \cos\psi, \quad \frac{dz_d}{dq_m} = -\sin\psi \qquad \text{(A6)}$$

By substituting Equations (A6) and (2) into Equation (A5) and by simple algebraic manipulations, we obtain Equation (4).

Since along the arc sector of q_m=constant, there are

$$\frac{dr}{d\theta} = 0, \quad \frac{dz}{d\theta} = 0, \quad \frac{d\theta}{d\theta} = 1 \qquad \text{(A7)}$$

Substitute q in Equation (1) by θ, then Equation (6) can be obtained by using Equation (A7).

The general equation of flow weight streaming into any curved surface S is

$$G = \iint_S \rho g \boldsymbol{W} \cdot \boldsymbol{n}\, dS \qquad \text{(A8)}$$

If the equation of curved surface S is $S(r,\theta,z) = 0$ (A9)
then the unit normal vector of curved surface is

$$\boldsymbol{n} = \frac{\nabla S}{|\nabla S|} = \frac{\frac{\partial S}{\partial r}i_r + \frac{\partial S}{r\partial\theta}i_\theta + \frac{\partial S}{\partial z}i_z}{|\nabla S|} \qquad \text{(A10)}$$

In the present situation, the curved surface S is quasi perpendicular intersecting plane, the equation of which is Equation (A2). Therefore its unit normal vector is

$$\boldsymbol{n} = \sin\psi i_r + 0 i_\theta + \cos\psi i_z \qquad \text{(A11)}$$

The vector form of relative velocity W is

$$\boldsymbol{W} = W_r i_r + W_\theta i_\theta + W_z i_z \qquad \text{(A12)}$$

Hence,

$$\begin{aligned} \boldsymbol{W} \cdot \boldsymbol{n} &= W_r \sin\psi + W_z \cos\psi \\ &= W_m \sin\alpha \sin\psi + W_m \cos\alpha \cos\psi \\ &= W_m \cos(\psi - \alpha) \end{aligned} \qquad \text{(A13)}$$

Besides, the area micro-element of this quasi perpendicular intersecting plane can be written as

$$dS = r\,d\theta\,dq_m \quad \text{(A14)}$$

Substituting Equations (A13) and (A14) into Equation (A8), we obtain Equation (7).

The density equations are the same as in report (7), therefore the paper does not list them.

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